

Kidwind Math Sample Problems – Answer Key

Swept Area Questions:

1. The blades are 45 meters long. The length of one blade is equal to the radius. $\pi \times 45^2 = \mathbf{6362 \text{ meters squared}}$.
2. The diameter is 60 meters, so the radius is 30 meters. $\pi \times 30^2 = \mathbf{2827 \text{ meters squared}}$.
3. $P = 1/2 \times 1.225 \text{ kg/m}^3 \times 6362\text{m}^2 \times 10^3 = 9,546,976.25 \text{ Watts}$ or about 9.55 Megawatts
4. $P = 1/2 \times 1.225 \text{ kg/m}^3 \times 6362\text{m}^2 \times 12^3 = 16,497,174.96 \text{ Watts}$ or about 16.5 Megawatts.
5. $P = 1/2 \times 1.225 \text{ kg/m}^3 \times 2827\text{m}^2 \times 20^3 = 13,852,300 \text{ Watts}$ or about 13.85 Megawatts.
6. There are many losses in drag and friction. The Betz Limit states that the best theoretical wind turbine can still only be 59% efficient.

Tip Speed Ratio Questions:

1. The length of one blade is equal to the radius. The distance travelled is equal to the circumference of the circle created by the blades. Circumference of a circle = $2\pi r$. So, $(2 \times \pi \times 4) = \mathbf{25.13 \text{ meters}}$.
2.
$$\frac{1 \text{ Minute}}{42 \text{ Revolutions}} \times \frac{60 \text{ Seconds}}{1 \text{ Minute}} = \frac{60}{42} = \mathbf{1.43 \text{ Seconds}}$$
 per revolution
3. Velocity = distance divided by time. $\frac{25.13 \text{ meters}}{1.43 \text{ seconds}} = \mathbf{17.6 \text{ m/s}}$
4.
$$\text{TSR} = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}} = \frac{17.6}{6} = \mathbf{2.93}$$
5. Optimal TSR = $\frac{4\pi}{n} = \frac{4\pi}{3} = 4.2$ —The blades are moving too slow
6. $2 \times \pi \times 60 = 377 \text{ meters travelled}$. $\frac{60}{12} = 5 \text{ seconds per revolution}$.
 $\frac{377 \text{ meters}}{5 \text{ seconds}} = 75.4 \text{ m/s}$. $\text{TSR} = 75.4/18 = 4.2$. This is the optimal TSR for this wind turbine.

Tip Speed Ratio Questions (continued):

7. $2 \times \pi \times 0.25 = 1.57$ meters travelled. 600 rpm = 10 revolutions per second. 1 revolution = 1.57 meters. **Tip speed = 15.7** meters per second.
8. 1 meter per second = 2.237 miles per hour. $9/2.237 = 4.02$ meters per second.
9. $TSR = \frac{\text{Tip Speed of Blade}}{\text{Wind Speed}} = \frac{15.7}{4.02} = \mathbf{3.91}$

Calculating Height Questions

1. $3/1.7 = x/164.333$ $x = \mathbf{290 \text{ feet}}$
2. Answers vary... $H/8 = x/500$ (H=student's height). So, if they are 5 feet tall, $5/8 = x/500$ $x = 312.5$ feet
3. With method 2, the vertical height of the tower is equal to the horizontal distance from where you stood to the base of the tower plus your height. $2.2 \times 111 = 244.2 + 5.75 = 249.95$ or about **250 feet**.
4. $7 \times 32.5 = \mathbf{227.5 \text{ feet}}$.
5. Answers vary.

Gear Ratios Questions

1. $1680/12 = 140$ The gear ratio is **140:1**
2. $2100/120 = \mathbf{17.5 \text{ RPM}}$
3. $600 \times 6 = \mathbf{3600 \text{ RPM}}$ $3600/790 = \mathbf{4.56 \text{ Volts}}$
4. Final gear ratio = $(1260/70) \times (70/14) = \mathbf{90:1}$. $90 \times 15 = \mathbf{1350 \text{ RPM}}$

Word Problems

1. $\$2,100,000/14,000,000 = \mathbf{\$0.15 \text{ per kWh}}$
2. $\$2,100,000 \times 10 = \mathbf{\$21,000,000}$
3. $\$15,750,000/\$2,100,000 = \mathbf{7.5 \text{ years}}$
4. $14,000,000 \times 0.9 = 12,600,000$... $2,100,000/12,600,000 = \mathbf{\$0.1667}$
or about 17 cents per kWh
5. $14,000,000 \times 0.9 = 12,600,000$... $12,600,000 \times \$0.15 = \$1,890,000$
per year. $\$15,750,000/\$1,890,000 = \mathbf{8.333 \text{ years}}$ (8 1/3).
6. $10 - 8.333 = 1.667$... $1.667 \times \$1,890,000 = \mathbf{\$3,150,000}$
7. $2.1 \times 0.6 \times 42 = \mathbf{59.92 \text{ MW average}}$
8. $59.92 \times 24 \times 365 = \mathbf{463,579.2 \text{ MWh per year}}$. $463,579.2 \text{ MWh} \times$
 $1,000 \text{ kWh/MW} = \mathbf{463,579,200 \text{ kWh per year}}$
9. $463,579,200 \text{ kWh} \times \$0.15 = \mathbf{\$69,536,880 \text{ per year}}$
10. $\$69,536,880 \times 0.05 = \mathbf{\$4,172,212.80 \text{ per year}}$

Coefficient of Power Problems:

1. $0.62 \times 0.59 = 0.37$ or about 37%
2. $50 \text{ volts} \times 20 \text{ amperes} = 1,000 \text{ watts}$. $1,000/3,900 = 0.26$ or 26%
3. a. $P = 1/2 \times 1.23 \times 2\pi(1) \times 12^3 = 6,677.3 \text{ Watts}$.
b. $12 \text{ volts} \times 33 \text{ amps} = 393 \text{ Watts}$
c. $C_p = 393/6,677.3 = 0.06$ or about 6%.

4. This man is probably a con artist! The fact that he is claiming that his turbine is 73% efficient is how you know. This is impossible since the Betz limit is 59%. Therefore, no wind turbine can exceed this efficiency! Also, it is a really bad idea to put a wind turbine on your roof. This can cause structural damage to your house and also create a good bit of noise due to vibrations. Tell the salesman to go away!

5. 25 is not a good tip speed ratio. Once again, this man is a con-artist! When the blades are spinning too fast, they appear like a solid wall against the wind. That reduces efficiency. Also, each blade creates turbulent air in its wake as it spins around. If the next blade arrives too soon, it will hit that turbulent air, which also reduces efficiency.